Code No: 182AR **R22**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year II Semester Examinations, January/February - 2024
ORDINARY DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS
(Common to CE, EEE, ME, ECE, CSE, EIE, IT, AE, MIE, CSIT, CE(SE), CSE(CS), CSE(AI&ML), CSE(DS), CSE(IOT), AI&DS, AI&ML, CSD)

Time: 3 Hours **Max. Marks: 60**

Note: This question paper contains two parts A and B.
i) **Part- A** for 10 marks, ii) **Part - B** for 50 marks.

- Part-A is a compulsory question which consists of ten sub-questions from all units carrying equal marks.
- Part-B consists of **ten questions** (numbered from 2 to 11) **carrying 10 marks each**. From each unit, there are two questions and the student should answer one of them. Hence, the student should answer five questions from Part-B.

PART- A

(10 Marks)

1.a) Write down the general form of a linear differential equation of first order. [1]
b) Give an example of a family of curves which is self-orthogonal. [1]
c) Define the Legendre's homogeneous differential equation. [1]
d) Find the particular integral of $\frac{d^3y}{dx^3} + 4\frac{dy}{dx} = \sin 2x$. [1]
e) Show that the Laplace transform of the unit step function $u(t-a)$ is $\frac{e^{-as}}{s}$. [1]
f) State the Convolution Theorem for Laplace transform. [1]
g) Define a vector point function give an example. [1]
h) Find the curl of $\vec{V} = e^{xyz}(\vec{i} + \vec{j} + \vec{k})$ at the point $(1, 2, 3)$. [1]
i) State Stoke's Theorem. [1]
j) If S is a closed surface and V is the volume of the region bounded by S then what is the value of $\oint_S \bar{r}.\bar{N}dS$ where $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$. [1]

PART - B

(50 Marks)

2.a) A bacterial population B is known to have a rate of growth directly proportional to B itself. If between noon and 2 PM the population triples, at what time, no controls being exerted, should B become 100 times what it was at noon.
b) Solve $x\left[\frac{dx}{dy} + y\right] = 1 - y$. [5+5]

OR

3.a) Solve $[1 + \log(xy)]dx + \left[1 + \frac{x}{y}\right]dy = 0$.
b) Find the particular member of the orthogonal trajectories of $x^2 + cy^2 = 1$ passing through the point $(2, 1)$. [5+5]

4.a) Solve $(2x+5)^2\frac{d^2y}{dx^2}-6(2x+5)\frac{dy}{dx}+8y=6x$.

b) Solve $\frac{d^2y}{dx^2}-y=x\sin x+x^2e^x$. [5+5]

OR

5.a) Apply the method of variation of parameters to solve $\frac{d^2y}{dx^2}+y=\tan x$.

b) Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+5y=0, y(0)=-3$ and $\frac{dy}{dx}=1$ at $x=0$. [5+5]

6.a) Find the Laplace transform of the Half wave rectifier given as

$$f(t)=\begin{cases}\sin\omega t, & 0<t<\frac{\pi}{\omega}\\ 0, & \frac{\pi}{\omega}<t<\frac{2\pi}{\omega}\end{cases}$$

b) Solve by using the Laplace transform $y''+2y'+5y=e^{-t}\sin t, y(0)=0,$ and $y'(0)=1$. Here $y'=\frac{dy}{dt}$. [5+5]

OR

7.a) Evaluate $L^{-1}\left\{\frac{e^{-s}-3e^{-3s}}{s^2}\right\}$.

b) Evaluate $\int_0^t e^{-t}\left(\frac{\cos at-\cos bt}{t}\right)dt$ with the help of Laplace transform. [5+5]

8.a) If $\vec{v}=\frac{x\bar{i}+y\bar{j}+z\bar{k}}{\sqrt{x^2+y^2+z^2}}$, find the value of divergence $(\vec{v})$.

b) Find the value of n for which the vector $r^n\vec{r}$ is solenoidal, where $\vec{r}=x\bar{i}+y\bar{j}+z\bar{k}$. [5+5]

OR

9.a) Find the divergence and curl of $\vec{v}=(xyz)\bar{i}+(3x^2y)\bar{j}+(xz^2-y^2z)\bar{k}$ at $(2,-1,1)$.

b) Prove that $(y^2-z^2+3yz-2x)\bar{i}+(3xz+2xy)\bar{j}+(3xy-2xz+2z)\bar{k}$ is both solenoidal and irrotational. [5+5]

10.a) A vector field is given by $\vec{F}=(\sin y)\bar{i}+x(1+\cos y)\bar{j}$. Evaluate the line integral over a circular path $x^2+y^2=a^2, z=0$.

b) Evaluate $\iint\left(yz\bar{i}+zx\bar{j}+xy\bar{k}\right)\overrightarrow{ds}$ over S, where S is the surface of the sphere $x^2+y^2+z^2=a^2$ in the first octant. [5+5]

OR

11.a) Use Green's Theorem, evaluate $\int(x^2ydx+x^2dy)$ over C, where C is the boundary described counter clockwise of the triangle with vertices $(0,0),(1,0),(1,1)$.

b) Apply Stoke's Theorem to find the value of $\int(ydx+zdy+xdz)$ over C, where C is the curve of intersection of $x^2+y^2+z^2=a^2$ and $x+z=a$. [5+5]

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